

WEB APPENDIX

Sarstedt, M. & Mooi, E. (2019). *A concise guide to market research. The process, data, and methods using SPSS* (3rd ed.). Heidelberg: Springer.

Levene's Test

How do we determine whether two populations have the same variance? The Levene's test (also referred to as the *F-test of sample variance*) attempts to answer this question using the following hypotheses:

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Levene's test compares the variability between the groups (numerator of the test statistic) with the variability within the groups (denominator): The higher the share of the between-group variation, the higher the value of the test statistic and, thus, the higher the tendency to reject H_0 . The test statistic F is calculated using the following formula:

$$F = (n - 2) \frac{\sum_{j=1}^2 n_j (\bar{y}_j - \bar{y})^2}{\sum_{j=1}^2 \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)^2}$$

with

$$y_{ij} = |x_{ij} - \bar{x}_j| \quad \text{and} \quad \bar{x}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} x_{ij},$$

$$\bar{y}_j = \sum_{i=1}^{n_j} y_{ij} \quad \text{and} \quad \bar{y} = \frac{1}{2} \sum_{j=1}^2 \bar{y}_j \quad ,$$

where n is the total number of observations and n_j the sample size of group j .

The null hypothesis is rejected if F exceeds the $(1 - \frac{\alpha}{2})$ -quantile of an $F(1, n-2)$ -distribution. We won't discuss the F -distribution in detail—you only need to compare the test statistic value with the tabulated quantiles of an F -distribution, which the significance level α (e.g. $\alpha = 5\%$), the number of groups (in our example, 2), and the number of observations n (in our example, 20) determine. Alternatively, you can calculate the exact p -value by using the Excel formula =FDIST(1.532,2,20), where the first value 1.532 is the F -value as calculated later and 2 and 20 refer to the number of groups and the sample size. You can then see whether the resultant p -value of 0.24 is smaller than 0.05 (which it isn't and you therefore find support for the null hypothesis).

To answer our research question of whether the point of sale display and the free tasting stand have the same sales volume (Chap. 6), we first have to determine whether both campaign types show a similar variability in sales. Consequently, we need to calculate the F -statistic in a few steps.

The total number observations n is 20, with 10 originating from each campaign type ($n_1 = 10$ and $n_2 = 10$). The mean sales of the point of sale display is $\bar{x}_1 = 47.30$ and the

mean sales of the free-tasting stand is $\bar{x}_2 = 52.00$. Table A6.1 shows the absolute values of the differences between the single observations and their respective group means:

Point of sale display: $y_{1j} = x_{i1} - \bar{x}_1 $	Free tasting stand: $y_{2j} = x_{i2} - \bar{x}_2 $
2.7	3.0
4.7	3.0
4.3	3.0
0.7	5.0
0.3	3.0
2.3	3.0
3.3	4.0
1.7	2.0
3.7	2.0
3.3	8.0

Table A6.1: differences in observations

Based on this tabulation, we calculate the group means as $\bar{Y}_1 = 2.70$ and $\bar{Y}_2 = 3.60$ by dividing the sum of the values of the two columns (27 and 36) by the number of entries per column ($n_1 = n_2 = 10$). Based on these results, the test statistic can be calculated and compared to the $(1 - \frac{\alpha}{2})$ -quantile of an $F(1, 28)$ -distribution as follows:

$$F = 18 \cdot \frac{10 \cdot (0.45^2 + 0.45^2)}{19.20 + 28.40} = \frac{72.90}{47.60} = 1.5315126(\dots) \approx 1.532$$

Using a significance level of $\alpha = 5\%$, the respective $F(1, 28)$ -quantile amounts to about 5.61, which exceeds the value of the test statistic F . Consequently, we cannot reject H_0 . We therefore have to use the pooled variance estimate when comparing the two groups, which Chap. 6 of the book describes.

Note that Levene's test can also be applied to check whether the variances of more than two groups are equal (or not):

For k groups: $H_0: \sigma_1^2 = \dots = \sigma_k^2$ vs. $H_1: \sigma_i^2 \neq \sigma_j^2$ for at least two groups i and j with $i \neq j$

$$\text{Test statistic: } F = \frac{n-k}{k-1} \frac{\sum_{j=1}^k n_j (\bar{y}_j - \bar{y})^2}{\sum_{j=1}^k \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)^2}$$

$$\text{Rejection region: } F > F^{1-\frac{\alpha}{2}}(k-1, n-k)$$